

What is the \mathbb{N}_0^{th}

Question on the

Test?

¹ It could be objected that keeping to these sequences is to omit infinite numbers of other sequences of ordinally indexed aleph-numbers from consideration. A detailed account of how this is “permissible” will be diverted until much later in this essay.

\aleph_{\aleph_0} is \aleph_0 -countable. Despite our knowledge of this set of facts² about \mathfrak{c} , the Gödel-Cohen results show that within ZFC, the question of the Continuum and its cardinality is not completely well-formed. By the **axiom of transfinality** (of which the local counterdeduction from cofinality, regarding the Continuum, is an instance), we might know much of what the Continuum's cardinality could not be, but never what it in itself must be. What appears, here, is a sort of Kantian antinomy of infinity and intuition. In naïve neo-Kantian terms, then, we would say that this issue is simply a matter of intellectual discipline, and that the Gödel-Cohen results testify to the need for, and the form of, this discipline, wherefore the negative deductions as from the axiom of transfinality by itself are sufficient, otherwise, to the task at hand (as there is no *absolute* task of proof at hand, in the end).

Because, "What does $\aleph_0^{\aleph_0}$ equal?" does not appear to be an indeterminate question in its own right, I, however, am going to go on an adventure through Cantor's paradise, to find out whether this mystery is not itself just the shadow of a more authentic—and therefore absolute—question.

First, I will call *the Continuum Hyperthesis* the conjecture that \mathfrak{c} is "probably"³ one of the cardinals in the transquadrant described from \aleph_0 through \aleph_{\aleph_2} , plus the first three alephs in the infinite sequence marked out by infinite numbers of glyphs per mark, i.e. "up to":

$\aleph_{\aleph_{\aleph_2}} \dots \aleph_2$

Note again that one number in this transquadrant has already been ruled out. The natural candidate for $\aleph_0^{\aleph_0}$'s value, for much of set theory's explicit history, has been \aleph_1 , since the difference between the rational and the irrational numbers (used in the very Diagonal

² In a related vein, although we do not know that $2^{\aleph_n} = \aleph_{n+1}$, we know that in ZFC, *ceteris paribus*, $2^{\aleph_\omega} < \aleph_{\omega_4}$ (see Saharon Shelah, "Cardinal Arithmetic for Skeptics"), or in other words that, if $\aleph_0^{\aleph_n} < \aleph_{\aleph_0}$, then $\aleph_0^{\aleph_{\aleph_0}} < \aleph_{\aleph_4}$.

³ I say "probably" inasmuch as the expression 2^{\aleph_0} indicates that the Continuum is the direct successor, the second successor, the successor by series, the second successor by series, or the successor of the first successor by series, and so on, of the first aleph-number. Now, granted, since $2^{\aleph_0} = 3^{\aleph_0}$ and so on, the presence of 2 in the base expression might be merely lending an illusion to the hyperthesis, as such. However, the case where $n = 2$, here, is the simplest other than the one for $\aleph_0^{\aleph_0}$, and this latter expression also indicates a direct successor (in the space of the relatively transfinite *simpliciter*), a successor of a successor, or so on and on, also.

Argument of Cantor's) seems successive, i.e. it seems that the concept of irrational numbers, and hence their infinity, is the immediate successor of the concept of the rationally infinite. Since no one seems to have ever clearly discerned any "kinds" of numbers larger than rationals but smaller than reals, this too testifies to there being as close to a minimum gap between \aleph_0 and \mathfrak{c} as possible. Yet the Continuum Hypothesis (that $\mathfrak{c} = \aleph_1$) is not necessarily accepted in general, anymore, on such an "intuitive" ground, since often enough such intuitions turn out to be masques placed over incorrect, if analytically determinate, attempts at proofs and conclusions.

If the question of $\mathfrak{c} = \aleph_x$ is, then, a real one, with an exact answer, it seems that it should result "from" a formula (**erotic transet**⁴), as in the Generalized Continuum Hypothesis, that $2^{\aleph_n} = \aleph_{n+1}$. Now the cardinality of the Continuum could be \aleph_1 even if the GCH is false. Let us use the aleph-case to illustrate how something *like* the GCH *should* be true, though, by saying that

$$\aleph_n^{\aleph_m} = \aleph_{f(n,m)}; \{n \neq m\} \rightarrow \{\aleph_n^{\aleph_m} \neq \aleph_m^{\aleph_n}\}$$

The formulaic illusion of the GCH by itself, then, is that it deals with the zero in the glyphset for the first transfinite cardinality as if it were the same as the kind of zero as is used in standard arithmetic. But the uses of the number zero, in either case, are inequivalent (conceptually), so it is not to be expected that the solution to $\aleph_0^{\aleph_0}$ is just \aleph_{n+m+1} , for example. —To be sure, to preserve the functional haecceity of the transcension-action, here, while accepting (for example) that $(\aleph_0)^{\aleph_0^{\aleph_0}} = \aleph_0^{\aleph_0}$, will require much finesse, ultimately, but I do not intend to get that far ahead of myself just yet. Instead, I am going to precisely reinterpret the semantics of the aleph-glyphs themselves, in terms of...

⁴ In other words, the axiom of transcension *just is* the axiom according to which there is *some* formula for going from some aleph-number to the power of itself or another, on to some "much" higher aleph-number in the sets of series—or even from going from glyphset to glyphset by means of some other operation than exponentiation (or tetration, etc.). (Note here that \aleph_0 is taken to be both a number *and* an operator, inasmuch as the cardinals listed on all sets of series are countably discrete, wherefore the first glyphset also signifies the entire order of set-series on every possible level. Moreover, that countable infinity is given, here, is by the **axiom of transfinity**, which is otherwise equivalent to the ZFC axiom of the sole constructible infinity.)

Metrographs and metagraphs

The idea, here, is that there is a simplest means of representing the infinite series of cardinalities, the staircases altogether and their constitutive step-sets. And the simplest initial means, as geometrically given, is to have $\aleph_0, \aleph_1, \aleph_2, \aleph_3$, etc. as equivalent to points on a discrete, though infinite, line. I.e. the first point on this line is assigned the same semantics as the first aleph-glyph, and by succession each point on the line is made to correspond to the successor glyphsets for \aleph_0 and its counterpart point. This first linear transfinite order (“level of infinity”) is a *metrographic* glyphset.⁵

The metrographic theorem is that the geometrical intervals in the construction of the given metrograph correspond to arithmetic intervals in the sets of the aleph-series. Therefore, here, $\aleph_0^{\aleph_0}$ is conceived of as a point representative of infinity, taken to the infinite power of such a point. Intuitively, this corresponds to some point in the general transquadrant heretofore constructed that infinitely succeeds \aleph_0 . By the deduction from cofinality, \aleph_{\aleph_0} is eliminated as the solution, now; but simplicity (inasmuch as $\aleph_0^{\aleph_0}$ is the simplest example of transcension) indicates that the next infinite successor to \aleph_0 (considered as a single point of actual infinity) is the one ascended to as such, i.e. interpreted in this (intuitive) way, $\aleph_0^{\aleph_0} = \aleph_{\aleph_1}$.

Strictly speaking, the above is nowhere near sufficient proof of the point(!) to be made. This will be provided below; for the moment, I would like to consider that, again, $(\aleph_0)^{\aleph_0^{\aleph_0}}$ would seem to be more than the Continuum, since it would be equivalent to $(\aleph_{\aleph_1})^{\aleph_0} = (\aleph_0)^{\aleph_{\aleph_1}}$ if the prior assertion is otherwise true, yet therefore in violation of the requirement that output functions be one-to-one for the local formula of transcension. However, in terms of the metrograph, the transit from \aleph_0 to \aleph_{\aleph_1} is also a shift in the actual **dimensionality** of the glyphsets, and this can in no wise be summarily discounted from the transcension formula:

$${}_d^g \aleph_n^{\aleph_m} = \aleph_f(d, g, n, m)$$

—where g is the number (from 1 to \aleph_0) of times the aleph-glyph is given in the cardinal mark, and d is the dimensionality of the glyphset itself. Now, the transit from countable to continuous

⁵C.f. Lawrence S. Moss, “Non-wellfounded Set Theory,” §1.3, “A *graph* is a pair (G, \rightarrow) , where \rightarrow is a relation on G (a set of ordered pairs from G). The idea is that we want to think of [*sic*] a graphs as *notations for sets*...”

infinity is tantamount to the conversion of a point into an infinite lattice of points, and then this lattice's integration as a single linear order in total: metrographically, that is, this is the case. Therefore, \aleph_{\aleph_0} corresponds to the disintegrated set of all points in the first aleph-series, whereas its successor is the integrated set of these points. Therefore, in the original, zero-dimensional case of $\aleph_0^{\aleph_0}$, there is a unique ascension to the second-dimensional linear set of \aleph_{\aleph_1} , and subsequent iterations of the otherwise identical ascension formula are applications to glyphsets that differ from the original case by a factor of d .⁶

Lastly: metrographs are examples of **metagraphs**, which are given with the **axiom of transcardinality**. This subject will be addressed more after the proof of $\mathfrak{c} = \aleph_{\aleph_1}$; for now, the signal understanding to be had is that metagraphics explicitly depends on the concept of erotetic transects mentioned earlier.

Dedekind's knife

Let us, then, refer to **erotetic powersets**. The idea is that, instead of speaking of a set of all subsets of a given set, we speak of a set of answers from which an erotetic function can be deduced without it being possible to answer to that function with the information in the occurrent answer-set.

If there are any "kinds" of numbers intuitively greater than the rationals but lesser than the reals, it would be numbers that are epistemic approximations, to greater and greater degrees (from \aleph_0 onward, to \mathfrak{c}), of the elements of the Continuum as such. Now we know of both Cauchy sequences and Dedekind cuts as ways to construct the reals from the rationals, and more broadly we know from throughout history that many better and better methods of constructing even individual real numbers (e.g. π) can be had.

Taking the notion of erotetic powersets together with the function of Dedekind cuts in mapping to the real numbers, I will say that the "kinds" of numbers contained in \aleph_1 up through \aleph_{\aleph_0} , that differentiate these cardinalities from the discrete first and from the Continuum second, are numbers that are increasingly less and less epistemically **implected** in the

⁶ For introductory technical reasons, the utility of the g -factor, here, will be assessed in a later section, before the more difficult quest through Cantor's paradise commences.

The modal index of the Continuum

$$X_0 : \diamond x^8$$

$$X_1 : \Diamond O x$$

$$X_2 : \Diamond \circ \Diamond x$$

$$X_3 : \diamond O \diamond O x$$

...

What “happens,” then, when an infinite set of these modal operators is reached?

[illegible]

⁸ This is to be read as “possibly x.” The circles in the subsequent series are to be read as “actually.” In modal logic, it is given that the possibility of x is equal to the possible actuality of x, and that the actuality of x implies the actual possibility of x. By Barcan’s formula, these equivalences are themselves equivalent (if not identical). Therefore, in modal space, it is possible to add on an infinite number of either of these modal operators, or both together in some order (or not), on to x, assuming either at the inception of the process. And the x itself can serve in a digital expansion of such sequences as x.010101010 ... and so on. (The Gödel arithmetic, here, assigns the value 0.1 to \aleph_0 and 1.0 to the additive element of the aleph-exponent in the simplest case. 0.1 is not, strictly, a classical decimal number but is the **transciprocal** of 1.0, where 1.0 = the unit line. Accordingly, the transit to ϵ is $0.1 + 1.0 = 1.1 = \aleph_{\aleph_1}$, whereas if it were $0.1 + 0.9 = 1.0$, it would be \aleph_{\aleph_0} .)

[illegible]

If this is true, it follows that $(\aleph_{\aleph_0})^{\aleph_{\aleph_0}}$ maps outside of the first transquadrant. For metrographically, it would be possible, here, to get to every series in the first transquadrant *from* the first series plus the local formula of ascension. To preserve the uniqueness-of-output for the aleph-tetrations, then, the metrograph would be power-mapped from the first aleph-glyphset on the first successor series to another order of these series, in fact the first successor of the next order, i.e.:

$$x_{i_1} x_{i_2} \dots x_{i_n}$$

⁹ Although too ethereal to deal with right now, this notion of ghost metacardinals will be given a clearer role in the cartography of the transet paradise, down this essay's road.

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The index of spheration

On the above kind of view, there is a general way to “describe” the interiors of the sets of numbers signified by these series of metroglyphs: there is a dimensional ground for the different “kinds of numbers” even if the real number line is equinumerous with the 3-plane and so on, for there still are problems involving higher dimensionalities that must be solved using sets of high cardinality. What these particular sets are will be discerned as knowledge of geometry advances, or anticipated set-theoretically to some extent: but they are there, “waiting” for us.

¹⁰ It is an open question, whether the problem of quantum renormality is equivalent to the reduction from \mathfrak{c} to \aleph_0 or something else. At issue is why the renormalization procedure fails to be workable with respect to gravity. My suggestion, here, will be that the process of renormalization might have multiple values such that it is also cardinally equivalent to the reduction from π_0 to $\hat{\pi}_0$ (or some similar critical point in the retrogress of cofinality).

\aleph_n -dimensional graphs and the \beth -numbers

Given the rules of glyphic construction available,¹¹ it seems possible to refer to metrographs whose dimensionality is not n -dimensional for $n = \aleph_0$ but higher cardinalities, indeed any cardinality represented directly or indirectly up until this point. From these, even more intricate metroglyphs can be derived, neverendingly.

Next, let us invoke the beth-numbers:

$$\beth_0 = \aleph_0$$

$$\beth_1 = \aleph_0^{\aleph_0}$$

$$\beth_2 = \beth_1^{\beth_1}$$

$$\dots \beth_{\aleph_0} = \tilde{\aleph}_1$$

Inasmuch as the beth-numbers represent the axiom of transcension's action¹² across the entire metrographic sequence so far evidenced, and inasmuch as \beth_n can be set to \aleph_0 , the infinite application of this axiom provides for an **erotetic series** in aleph-space, namely the $\tilde{\aleph}_n$ -series. That is, it is possible to “go to” cardinals higher than any whose glyphsets are constructed in the metrograph deduced by the simplest standard method from \aleph_0 . Each range of these cardinals spans an entire infinite set of ascensions, as such.

Now, again, what if we “go to” $\tilde{\aleph}_{\aleph_0}$? Purely by use of erotetic powersets, here, it appears that an infinite series of infinite numbers of levels of transfinity might be characterized, even between types of metrographic space. —And it is also possible, by the axiom of transfinality, to look at this journey as if in a mirror (of cofinality): $\tilde{\aleph}_1$ might be reported as the first cardinality that it takes an infinite number of *descensions from*, to get “back” to \aleph_0 (in which event $\aleph_0 = \tilde{\aleph}_0$).

¹¹ E.g. $\aleph_{\aleph_{\aleph_{\aleph_{\dots \aleph_0}}}}$ can be represented as $\aleph_0 \aleph_0$; however, note that $\aleph_1 \aleph_0$ is inadmissible (as the aleph-glyphs, when given, are discrete).

¹² Note that the classical beth-numbers are not exactly so defined, but the equivalence $\beth_0^{\beth_0^{\beth_0^{\dots}}} = \beth_1^{\beth_1^{\beth_1^{\dots}}} = \beth_2^{\beth_2^{\beth_2^{\dots}}} = \dots = \beth_n^{\beth_n^{\beth_n^{\dots}}}$ guarantees that the definitions ultimately overlap. (This infinite equation is validated by the known arithmetic for aleph-zero and the Continuum.)

The κ -erotetic landscape

Let us define a glyphset such that:

$$\tilde{x}_0 > x_\kappa > \tilde{x}_{\aleph_0}$$

I.e. from \tilde{x}_0 , there is an infinite series of infinite sets of descensions “towards” x_κ , and an infinite series of ascensions “from” \aleph_0 to the “same” κ , which is however virtual—this is the **ghost-heart** of the **nexus of metafinality**.

Not only the transfinal axiom but the transcardinal one, must be invoked to define the glyphset in question. In this context, we invoke the metagraphs of these axioms’ formal logic. But this is not just propositional logic: it is a unity of erotetic, assertoric, and prescriptive logic as the units of pure transyntax. And the logic of the system can in this light be interpreted as a joint deontic-modal transyntax, where modality circumscribes any two of the three at once, and deontic functions circumscribe all three at once.

Kant’s second theorem (here) is that the supersuccession to the end of transcension—“the set of all possible sets” or **absolute infinite**—is an erotetic action-space, i.e. the multitude of problems of infinite synthesis in transcendental dialectic are configured as *the* problem of absolute transcension.

—But deontic logic can encode actually infinite imperatives.¹³ Therefore, there is a series of levels of sets of κ “at the edge of” the set-theoretic n -verse that are **deontic cardinals**, starting (now) with \mathfrak{D}_0 . In the mirror of cofinality, these otherwise reflect the very first metrograph’s transet = $\{\tilde{x}_0, \tilde{x}_1\}$, only the deontic cases converge “towards” absolute infinity, so to say (just as simple transfinite cofinality converges towards \aleph_0 , and from there deeper to zero *per se*).

¹³ An example of this would be in the definition of the supererogation operator-space, which gives an infinite sequence of transobligated actions, each of which has more cardinal value than the prior element in the sequence, down to the basic case. (And in relation to the Moretti sequence here mentioned, each point on each graph, over and above those required for the other basic operators, corresponds to a form of “transerogated” action.) Moreover, G. E. Moore defines deontic cardinal value as that which ought to exist for its own sake; but this function “ought to exist for its own sake” does not admit of being added up in the finitary way, wherefore Moore’s consequentialist mathematics of ethics does not go through. I.e. 1-ought + 1-ought \neq 2-ought. The set of the two 1s does not “deserve” to exist more than either individual set. This suggests that deontic cardinality is no less than always \aleph_0 , but as the imperative logic of the system would have it, this cardinality is different from that one. At any rate, $\aleph_0 + \aleph_0 = \aleph_0$. [Elsewhere: bring up the Korsgaard cartographic theorem and O’Neill’s transparticular judgment.]

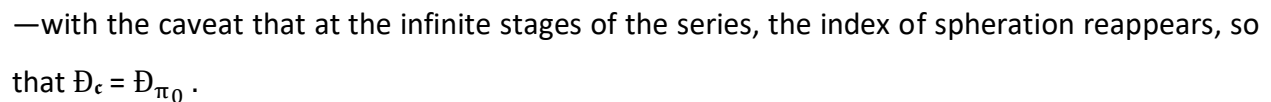
$\underline{+}x$ to, "Do refer to x ,"¹⁴ and $\underline{-}x$ to, "Do not refer to x ," this determination is *via*:

$$\mathcal{D}_1 : \underline{+\neg X}$$

$$\dots \mathbb{D}_{\aleph_0} : \underline{\overline{++++++}} \dots$$

The initial metrographic image of the deontic cardinals is naturally given by Alessio Ci's n -dimensional series of deontic graphs:

Moretti's n -dimensional series of deontic graphs:


$$\mathbb{D}_{\mathbb{N}_0} \rightarrow \emptyset_{\mathbb{N}}, \emptyset_{\mathbb{N}} = \begin{cases} 0_0 \\ 0_1 \\ 0_2 \\ 0_3 \\ 0_4 \\ \dots \\ 0_n \\ \dots \end{cases}$$

¹⁴ Allow \Rightarrow to stand for the **imperative of inference**, which is, “Infer y from x,” or, “From x, infer y.” This is a simple imperative operator, alongside the “why”-operator (discussed later). [Add in commentary on free will/chance/randomness.]

This series does not converge at absolute infinity, however: this convergence requires a set of series each of which is finite by itself but relatively finite *and* relatively infinite in relation to the others.

Intuitively, then, let us partly signify the apex-staircase as:

$$\begin{array}{r}
 1_0 \\
 2_0, 2_1 \\
 3_0, 3_1, 3_2 \\
 4_0, 4_1, 4_2, 4_3 \\
 5_0, 5_1, 5_2, 5_3, 5_4 \dots \\
 \dots 0_0, 0_1, 0_2, 0_3, 0_4, 0_5, 0_6, 0_7, 0_8, 0_9, 0_{10} \dots
 \end{array}$$

—wherefore 1_0 is at the “final edge” of the relatively infinite. Note that every level “above” zero-aleph is externally finite: this is where the unique successor relationship appears between \aleph_n numbers, as a level-theoretic series *simpliciter* (so that succession on each level after zero-aleph is finite). Moreover, there is therefore no exact succession from any 0_n to any apex-level n_n , e.g. 0_{15} does not “go to” $15_0 \dots$ by the axiom of transcension.

The axiom of transcardinality

By virtue of this axiom, the absolute infinite (the transet of this) is known as a **metafinite** reality. It completes the series of the absolute finite, relatively finite, and relatively infinite (though the problem of **antifinity** is not solved, here). This is an erotetic adduction: the absolutely infinite question is given, but only transfinally is the answer infinitely given (in deontic aleph-space), so that an illicit “transcendent” knowledge is not else given in itself.

Granted the use of the axiom of noncontradiction in the foundations of set theory, transcardinal equality must be accounted for in order to ostensibly present the value of the axiom of transcardinality in turn. Now, metagraphically, noncontradiction can be expressed as both:

$$(A) \neg(X \wedge \neg X)$$

$$(B) \neg(X \wedge \neg X)$$

The (A)-definition is assertoric, saying, “There are no true contradictions.” But the (B)-definition is deontic: “**Do not infer contradictions.**”¹⁵ Now the erotetic function, “Why not infer contradictions?” adverts back to the propositional grounds of the (A)-definition, but this is proven not by building up from the identity of conjunction to that of negation and then disjunction, but instead proceeds from the ability to ask disjunctive questions themselves.¹⁶

The reason for bringing this subject up here and now is to address Cantor’s attempt to interpret the possibility of absolute infinity as “inconsistent multiplicities.” One clear example of this concept is the transet of logical explosion: if anything were infinite “in every way,” would that not be the case? —Yet a logical explosion is also absolutely infinitely *false*, so we must search for a different solution to the question of noncontradiction and its axiom.

Although Gödel’s theorems on consistency and completeness proofs refer to the concept if the liar paradox, the correspondence is not quite one-to-one. However, recast in the light of erotetic logic, these theorems apply to the general case of absolute proof such that the erotetic solution to the liar paradox spells out the final limit on proof by equiconsistency. — That solution is given first by representing this erotetic sequence:

1. *This sentence is false.*
 2. *Is this sentence false?*
 3. *No, this sentence is not false.*
- ∴ This sentence is true.*¹⁷

¹⁵ In terms of the \Rightarrow -operator, the coherence principle might be spelled out as $\forall s((s = A \wedge \neg A) \pm \neg(\rightarrow s))$, “For all s , if s is, ‘ A and not A ,’ do not infer s .”

¹⁶ Though L. J. Brouwer’s theory of “free subjective creation” as the source of mathematical facts is true, so is *ante rem* realism: the “Platonic Form” of free transconstruction is the reality in question. For an exact provision of this idea, see Elijah Millgram, “Practical Reason and the Structure of Actions,” §4, re: the notion of “Intendo.” (C.f. Korsgaard’s analogy of deontic cartography (“Reason and Constructivism in Twentieth Century Moral Philosophy”) or O’Neill on insubsumptive judgment (Constructions of Reason). These authors recapitulate Prichard’s thesis: “We can have an obligation to do an action long before the action is done. If so, the obligation cannot be a feature of the action, since it exists before the action does. It must be a feature of something else, if it is a feature of anything, and the obvious candidate is the agent” (Jonathan Dancy, “Harold Arthur Prichard,” §3. These all reflect the fact that deontic cardinality sums incommensurably with the type of summation in the other metrographs.) — Now, although Brouwer is not wrong in general, the intuitionistic rejection of the axiom of exclusive disjunction is wrong, because this axiom is known by erotetic adduction either **before** the other logical axioms (of negation and identity under conjunction) or **simultaneously** with them.

¹⁷ For more on the subject of the “honest sentence,” see Wojciech Żetaniec, “Why the ‘veridic’ is not any better than the ‘liar’.”

Likewise, {"The next sentence is true," "The previous sentence is false"} is dissolved by its erotetic context: "Is the previous sentence false?" does not match to, "The next sentence is true," because it is not an assertion but a question.¹⁸ Since the liar sentences default on erotetic semantic value,¹⁹ then to admit them as true would be tantamount to unquestioningly accepting them, i.e. as axioms of truth in itself. But in this essay, no claimed axiom is to go unquestioned: where the epistemic regress of assertoric deduction ends, erotetic adduction starts, wherefore inasmuch as the abstract proposition corresponding to the liar sentence is void of erotetic force, the liar sentence does not possibly refer to any concrete truth.

Erotetic functions can be interdefined between erotetic and prescriptive logic as **epistemic imperatives**. Note: this is not an absolute reduction, as the concept of the epistemic adverts to pure erotetic concepts. Now Fitch's paradox of knowledge transcendently reflects the unprovability stipulation in Gödel's incompleteness doctrine, so that the mathematical imperatives (of proof) *just are* the erotetic powerset operation. I.e. it is the erotetic implexion of our set-theoretic knowledge that makes eternal transcension possible in the first place.

Therefore, the absolute infinite is not "just" the erotetic powerset of all other sets (or 1_0 or any other transfinite cardinal whatsoever). Our application of the powerset concept is restricted to the "interior" graph of the diamond of metafinity, which though great is not absolutely ultimate by itself as such. In fact, 2_1 is not the powerset of 2_0 , and so on. The apex-glyphs' transcardinal rank is assigned a uniquely finitary order, i.e. the indexical finitude of κ_n on each apex-level maps back, on the diamond, to the absolutely finite: and this is the case inasmuch as the absolute infinite is not contrary to finite reality, but is absolutely equiconsistent with this.

¹⁸ Take the loop, "A: B is true; B: A is false." Now A is true if and only if B is true. So A = B is true. And B is true if and only if A is false. So, the loop in total can just be rewritten as, "A: A is false," and, "B: B is true." In other words, the liar and the honest sentences are logically interchangeable under erotetic disquotation.

¹⁹ The liar-indexical *does* correspond to a sentence with a natural usage: consider someone pointing at a piece of paper with some false sentence, e.g., " $2 + 2 = 7$," on it. If said someone says, "This sentence is false," while as such pointing, "This sentence" refers to, " $2 + 2 = 7$." Therefore, we must be careful not to say that the liar-indexical is completely "meaningless."

The antiset

Metagraphically, that which is not finite and that which is antifinite, are different. The set-theoretic invocation of this logic is: the **antiset**, $\Omega_{\#}$, is the contrary of the true axioms, i.e. it is as such the contrary axiom itself. But the first are the axioms of transconstruction. So the second is the **axiom of destruction** instead. Metaphysically, this is equivalent to action that negates construction from the correct precepts, to corrupt or to destroy what is constructed.²⁰

Moreover, the axiom of the antiset would be identical to the transet of logical explosion. This is given by such an explosion being absolutely self-contrary: it is unrestricted contrariety *per se nota*. Therefore, however, the antiset is not absolutely infinite, because the transet of antilogic is not an existent set (though it does exist purely as an erotetic *transet*, otherwise noted): or, by the interposition of the correct axioms and that of destruction, the negation of the axiom of destruction is equivalent to a transcendental proof of the axioms of transconstruction. QED

Finitary and infinitary axiom schematics

To sum up the foregoing, let us define a transet of erotetic maps from the axiom scheme of the metafinite order.²¹ There are fifteen unit cases of one to four of the axioms being applied, and given the context, there are eight infinitary cases of application (paired with each unit case that includes the axiom of transfinity). Since the antiset is generally the contrary case, its applications are just generally contradictory towards the others, so there are fifteen $\Omega_{\#}$ -elements in the schematic transet. Lastly, there is a zero-case where the constructive axioms are not applied (and the anticase for the zero-case is equivalent to the constructive scheme).²²

²⁰ Corruption is to subtract a substantive predicate from something that exists, whereas destruction is subtraction of existence itself. But since on this (metagraphical) level, essence and existence are corollaries, corruption reduces to a form of destruction (of substantive predication). Accordingly, there is only one axiom for the antiset. In “large cardinal” kinds of terms, this axiom would be something like, “This is the set that does not exist” (which adverts to the circle of the liar sentences, as will be demonstrated shortly).

²¹ This is “why” there are four axioms, by transcendental correspondence with the diamond of metafinity.

²² In this case, zero is not quite the classical empty set. It is exactly defined as the set to which no other elements have been added, the axiom-proviso being that (in this context) the application of the axioms of set theory is what adds elements to this set. (By comparison, then, the successors of zero are the juxtaposition of the absolute finite (via the axiom of transcardinality, which gives us the metafinite order) with the possibility of zero, but they are not understood originally as successors of zero. This reflects the fact that almost no one in the history of the world has

The §-numbers

The **section numbers** map the ghost-heart of the metafinite nexus, with the κ -erotetic landscape, to the diamond of metafinity. Since there is no actual particular cardinal that is “exactly between” \aleph_0 and \aleph_n as such, we instead take the first four x-like intervals in the landscape and map them to the series of finite and infinite absolute relativity. The transfinal intervals ascend from \aleph_0 and descend from \aleph_n , to the ghost-heart, which “occupies” the interval of absolute infinity on either “side.”

Accordingly, the problem of the neverending powerset sequence is recapitulated not at the edge of the apex of transfinality, but at the imaginary “center” of the sequence. Since the ghost-heart can never be directly accessed by ascension or descension, and indeed does not correspond to any specific cardinality in any possible series at hand, it is a §-number that makes use of the concept of absolute infinity possible in particular, but uncountably so.

$$\S|\aleph_0 - \tilde{x}_1|\S|\tilde{x}_1 - \tilde{x}_2|\S|\tilde{x}_2 - \tilde{x}_3|\S|\tilde{x}_3 - |\{\S\sum_x\}| - \tilde{x}_3|\S|\tilde{x}_3 - \tilde{x}_2|\S|\tilde{x}_2 - \tilde{x}_1|\S|\tilde{x}_1 - \emptyset_\aleph|\S$$

—Therefore, the finitude of the number of sections as such allows us to use the role of the absolute finite, in the total action from the metafinite diamond, to relate particular cardinalities to the overarching structure of the relatively infinite.

Transfinal and antifinal operators

To justify the intricate construction of the apex-nexus, let us refer to the arithmetic of the last staircase. Now since the infinite ascending series of successive conjunction operators provides for the operational erotetic form of the series of absolute transcension, the zero-aleph level-set admits of definition from a **transfinal operator**. In other words, the apex-numbers are those that satisfy the basic equations of what is here known as the **heart-operator**, \heartsuit :

$$1_0 \heartsuit 1_0 = 1_0$$

$$n < 2_n : n_0 \heartsuit \dots \heartsuit n_{n-1} = 1_0$$

By implication, there is a special equivalence between all hypordinate transects of the last staircase as such: the proof of this fact is that any given metafinite cardinal indexed there is

ever actually proceeded from knowledge of the empty set, as such, to knowledge of all the other numbers (as such).)

\aleph_0 -steps from zero-aleph, although each is also (once reached) differentially finite in distance from 1_0 .

The antiset also has an operator; the glyph used here is \nexists , whose fundamental equations are:

$$[\Omega_1 \nexists \Omega_1 = \Omega_0; \Omega_0 \nexists \Omega_0 = \Omega_1] : \{\Omega_0 \nexists \Omega_0\} \nexists \{\Omega_0 \nexists \Omega_0\} = \Omega_0$$

$$\{\Omega_a \nexists \Omega_b\} \nexists \Omega_c = \Omega_{-1}$$

The antifinal use of this operator cannot “destroy” the apex-staircase, but it does take one from a given step on the staircase, “down towards” zero-aleph. The $a:b:c$ equation is that which makes use of antifinality completely: it is these ultimately antifinal glyphsets that reduce the index of actual transcension given at some juncture on the apex-staircase.

Let us introduce (with some spontaneity) the assertion that some sets can be **coidential transets**, or **cosets**. For example, we will state:

$$2_0 \heartsuit 2_1 \approx \S_Z \heartsuit 1_0$$

The second expression is left unevaluated, the transfinal operator used as are operator glyphs in ω -notation. (At least, it is unevaluated in this context: the idea of a **chromatic index** might allow a resolved construction, here.) As such, then, 2_n is the coset of the ghost-heart with the apex of transfinality (under the transfinal operator)—whatever this means, in the end.

The transconstruction of the autaset

If $\pm a$ = “Do a ,” then the question, “Do a ?” and the question of a , are also, “Why do a ?” and there is a recursive operator, “Ask why,” wherefore there is the adjunction, “Why ask ‘why’?” Which, then, here, as the **transquest**,²³ is signified:

$$\pm_{i_0} a: \text{“Ask why?”}; \pm_{i_1} a: \text{“Why ask why?”};$$

$$\pm_{i_2} a: \text{“Why ask, ‘Why ask why?’” ...}$$

$$\pm_{i_{\aleph_0}} a: \text{“Why ask ‘why ask ’ why ask... why ask why ’ ...’ ...?”}$$

²³ A *quest* can be thought of as a game-theoretic sequence of ascension or descension on the last staircase, by transfinal or antifinal operation. The apex-number assigned to an individual person at a given time, determines the *length of the quest*, which is equal to the number of κ from the assigned space to the conclusion of the apex. Therefore, the transquest is the question that gives the quests their essence in the limit.

Now, a can take the infinite supererogation sequence, which is of cardinality \beth_0 , and in the \pm_{\aleph_0} case we have an infinite deontic sequence $\beth_0^{\beth_0} = \beth_1$.²⁴ Now \beth_1 is the Continuum (by definition). There is, therefore, a unique element from the set of the Continuum that is associated with deontic advergence. This individual, finite number is an irrational number; in fact, it is transcendental: it is not keyed to from any normal algebraic procedure but is elected by free agency *per se nota*.

Pure agency is pure transpossibility. The infinite subset of possibility in the index of the modal continuum is therefore paired with an infinite, unique sequence of actuality. Due to the haecceity requirement on free actions, it follows that it is a singleton of the Continuum that is mapped to from the \beth_1 -order in deontic logical space. Now although this number comes from the Continuum, it has only countably many digits in its subspansion. Nevertheless, the assignment of digits to our epistemic implexion of this expanse will therefore take all eternity, for all we know.

For reasons of style, let us set the glyphset of this **autoset** to:

$$\mathfrak{A}_E \cdot 2_0 2_1 \aleph_0 \aleph_{x_0} \aleph_{x_1} \dots \S_\Sigma 1_0$$

—in which each glyphset represents a finite digit, reducible to some arbitrary binary value, and in which the association between the cardinals listed is not successively ordered but elective: this maps a unique sequence of ascensions and descensions within the ambit of relative infinity as such. The integer value of \mathfrak{A}_E is not known within mortal time; the 2_n and \aleph_0 digits are affixed, as are the virtual “final” digits. But which number is actually referred to, here, is unknown: we know there are different digits but not how to evaluate what they reduce to, from their alephic semiosis.

Allow, then, the Gödel sequence

$$f + \mathfrak{A}_E + F_0 + F_1 + F_2 =$$

$$\{\mathfrak{D}_E: \{a \mathbb{C} b\} \stackrel{\text{def}}{=} \text{“}a \text{ dreams about } b\text{”}\}; \left\{ \begin{array}{l} a \odot b : \text{the empty dream} \\ a \star b : \text{the nightmare operator} \end{array} \right.$$

... where f and F_n are finite numbers that are Gödel numbers for the five elementary deontic operators, so that the dream-operator is the interposition of the summary Gödel number with the operator-scheme in play.

²⁴ Alternatively, the presence of the second beth-number, in the transquest, can be given from the deontic continuum. However, there are to be five beth-numbers indexed in the transquest, so that at least one finite element from each of the five can be via a form of Gödel arithmetic (see below) computed to an analogy of the transfinal operator. So we look for a sequence that gives us five beth numbers, if possible. Now the, “Why ask why?” sentence can also be disjoined over the entire countable series of Moretti graphs, since each graph maps its own operators, for an infinite number of possible operators. And each of these operators can sustain the alternation to the Continuum, from the countably infinite \pm operation for each. So the five beth-numbers accessed include one represented by $\beth_n^{\beth_n^{\beth_n}}$, although as a stage in a tetration of others also.

Now dream-operations can be infinitely stacked or interleaved (“dreams within dreams” under operator iteration), giving us:

$$\{\{a \subset \subset \subset \dots b\}; \{a \star \star \star \dots b\}\} = \\ \{\{a^{\aleph} \subset \supset b\} \vee \{a^{\aleph} \star \supset b\}\} \vee \{a^{\aleph} \{\star, \subset\} \supset b\}$$

The infinite nightmare sequences do not Σ -reduce to a pure dream-sequence. They are like the infinite modal sequences: in fact, they are **the nightmare of antimodality**. This is the (false) concept, via the antiset, of modality as devolving upon not the $\Diamond x$ -semantics first but $\neg \Diamond x$, i.e. the antipossible instead of the possible. So there is an antifinite conversion not only from finitude-zero to the destruction of actuality as $a \nexists b$, but to the following perversion of transordinate necessity:

$$a^{\aleph} \{\star, \subset\} \supset b \approx \left\{ \begin{array}{c} \Diamond x, \neg \Diamond x, \Diamond \neg x, \neg \Diamond \neg x \\ \downarrow \\ \nexists \Diamond x, \nexists \neg \Diamond x, \nexists \Diamond \neg x, \nexists \neg \Diamond \neg x \end{array} \right.$$

Although these facts are difficult and tenebrous to consider, they are facts, to which we will return shortly.

The transquest ♡-sequence

For reasons of the section-numbers and the ghost metacardinals in general, and by virtue of the coset relation, in the light of empyrean summation, there is a “secret” heart-operation with an exact construction of particular epistemic value.

Now romantic correspondence between two individuals is uniquely one-to-one. As a metafinite estate, romantic idealism can be felt to four and only four degrees: the absolute finite, the relative finite, the relative infinite, and the absolute infinite. On the apex-staircase, romantic pairs are pairs of meta-elements from even-numbered transets, while friendship is pairing from odd-numbered transets. E.g., 2_0 and 2_1 go together, are parts of the “romantic coordinates” of two individual people. (Intuitively, each person has two coordinates, so any two people have four coordinates between them, and no coordinates are identical; and it is from these coordinates that any free agent assigns values to the digit-spaces of the autoset glyphset.)

One person can, therefore, fall in absolute love with someone else, and this is the apex-example of romantic metafinity. In fact, this is the final state of transit through romantic ideality: in other words, it is only possible to actually fall in love four times total, and the absolute case is indelible.

The pure relationship between that ideal and the apex-staircase is known in that the **isolate limit** is two: i.e. being isolated is being alone. Moreover, the index of personal agency is haecceitic, matched uniquely to one and only one apex-number as such, for everyone differently: this corresponds to romantic haecceity, which is love as a unique relationship

between a and b , justified by haecceitic reason and not aretaic rank. No person is replaceable in deontic space, and so romantic irreplaceability is a transcopy of deontic value. —And the **two empyrean attitudes**, which are **love** and **saudade**, are mapped to each other in erotetic space: the attitude of saudade is a question to which love in itself (not only the attitude) is the answer. Allegorically, love and saudade themselves “are in love with” each other.

By virtue of the axiom scheme in general, it can be known that there are only four infinitary cases of the transfinal operator. The last goes with the maximum of all the axioms together, in the infinitary case. As noted, the 2_n glyphsets, for example, are already “in play” in the game-theory of the apex-staircase, via the \heartsuit -operator, as are a few others (e.g. \aleph_0 , \aleph_Σ , 1_0). Let us collectively refer to these, in the aleph-catalogue, under the heading of $\langle\alpha, \beta\rangle$ and only have the ultimate heart-sequence for some arbitrary two other pairs of cardinals:

$$\left\{a \heartsuit \left\{b \subset \left\{a \heartsuit \left\{b \subset \dots\right\}\right\}\right\}\right\} \heartsuit \left\{a \subset \left\{b \heartsuit \left\{a \subset \left\{b \heartsuit \dots\right\}\right\}\right\}\right\} \\ \approx \\ \left\{x \heartsuit \left\{y \subset \left\{x \heartsuit \left\{y \subset \dots\right\}\right\}\right\}\right\} \heartsuit \left\{x \subset \left\{y \heartsuit \left\{x \subset \left\{y \heartsuit \dots\right\}\right\}\right\}\right\}$$

... which might be styled more compactly as:

$$\sum_{a,b}^{\heartsuit \subset} \approx \heartsuit \approx \sum_{x,y}^{\subset \heartsuit} \leftrightarrow \{x, y \neq \{\alpha, \beta\}\} :: \{a, b\}$$

However, by way of their haecceity, these interpositions are **all** equivalent.²⁵ This universal state of affairs is the fact of **absolute isoquence** in the context of metafinity.

Now, for romantic ideality to adverge within the domain of infinite forces as exalted as all of these—indeed, all of them whatsoever—establishes that it is itself a force of absolute value. The negation of this ideal, via the axiom of destruction, corrupts our knowledge of such advergence: the antimodal nightmare tries to “terrify” us into believing that the concept of evil is the logical prerequisite for the definition of the right, whereas the concept of evil only goes with prohibition in deontic operator-space: there are, that is, less forms of evil than forms of

²⁵ Note here that this Σ -case takes the dream- and transfinal operators together, that is, interleaved: this mirrors the nature of the \heartsuit -operator overall, as a reflection of the $+$ -additive and \pm -deontic glyphsets over the value of finite duality. The central Σ -heart is isoquent to the ghost-heart, and aleph-zero appears in the totality of the dreamheart-sequence.

right and good, in deontic logic alone, and the negative case is a successor from negation in general plus the local (operational) context.²⁶

The Test

If mathematics involves an absolute game, **the** Game, and if there is a “score in the Game,” then there is also a score on **the Test**. This is the transquest again, but in itself. So infinite numbers of other questions can arise, here and everywhere else in Cantor’s paradise. For example, the mathematical relations of color itself can be used to qualify the index of glyphsets in general, as can a sound index be assigned thereto. This would be a unique pair of differential glyphmarks with a construction from the pure system, but how they would actually work, I don’t know. It would be tied to a **redemptrix sequence**, but what that is I will delay remarking on more, until the sequel.

Wherefore is the answer to the question, “What is the \aleph_0 th question on the Test?” that **all** of those questions are, together?

²⁶ In other words, the deontic conditions of metrographic transconstruction are inescapable: invoking the form of destruction in action and thought results in the destruction of mathematical knowledge because destructive imagination intrinsically contradicts the formation of the aleph-series in our *a priori* glyphsets.

The *omega-grid* of the aleph-numbers is the standard notation for the order to the universe of sets. Rather than using iterations of the aleph-glyph to express the transets of levels of infinity, it uses ordinals defined relative to ω :

$$\aleph_0 \aleph_1 \aleph_2 \aleph_3 \aleph_4 \aleph_5 \aleph_6 \aleph_7 \dots$$

$$\aleph_\omega \aleph_{\omega+1} \aleph_{\omega+2} \aleph_{\omega+3} \aleph_{\omega+4} \aleph_{\omega+5} \aleph_{\omega+6} \aleph_{\omega+7} \dots \aleph_{\omega+\omega} \dots$$

$$\aleph_{\omega+\omega+1} \aleph_{\omega+\omega+2} \dots \aleph_{\omega+\omega+\omega} \dots \aleph_{\omega+\omega+\omega+\omega+\omega+\omega+\omega+\omega+\dots+\omega} \dots$$

... eventually inclusive of glyphsets like $\aleph_{\epsilon \in \epsilon \dots \epsilon_0}$, and so on. Now, the discrepancy

between cardinal and ordinal arithmetic in general, and the unwieldy, operant expansions of the omega-glyphsets, makes the use of the above notation questionable as far as expressing transet-levels goes. Notably, although

$$\aleph_0 \aleph_1 \aleph_2 \aleph_3 \aleph_4 \aleph_5 \aleph_6 \aleph_7 \dots = \omega \ \omega_1 \ \omega_2 \ \omega_3 \ \omega_4 \ \omega_5 \ \omega_6 \ \omega_7 \dots$$

... there is, however, no such thing as an aleph-glyphset counterpart to $\omega + 1$, for example—or if there is, this is the same as \aleph_1 , even though $\aleph_{\omega+1} \neq \aleph_{\omega_1}$.

With the above schematics at hand, we can check the glyphsets introduced in this essay by reporting them as instances of the axioms being applied either in the unit or infinitary cases:

Transfinity — Transcension — Transfinality — Transcardinality
Transfinity, transcension — Transfinity, transfinality — Transfinity, transcardinality
Transcension, transfinality — Transcension, transcardinality — Transfinality, transcardinality
...
... \aleph_0 -cases...
[table to be completed]

$${}^{\mathfrak{s}}\Sigma^{\dots}1_0$$

$$2_0 \quad 2_1$$

$$3_0 \quad 3_1 \quad 3_2$$

$$\pi_0 \dots \beth_1 \dots \tilde{\mathfrak{X}}_2 \dots \kappa_{\mathfrak{S}_{\Sigma}} \dots {}^{\mathfrak{d}}\mathfrak{D}_3 \dots 0_4 \dots 5_0 \dots$$

$\aleph_0 \aleph_0 \qquad \aleph_0 \aleph_1 \qquad \aleph_0 \aleph_2 \dots \tilde{\mathfrak{g}}_n \dots$					
${}^3\aleph_0$	${}^3\aleph_1$	${}^3\aleph_2$	${}^3\aleph_3$	${}^3\aleph_4$	${}^3\aleph_5$
${}^2\aleph_0$	${}^2\aleph_1$	${}^2\aleph_2$	${}^2\aleph_3$	${}^2\aleph_4$	${}^2\aleph_5 \dots$
${}^1\aleph_0$	${}^1\aleph_1$	${}^1\aleph_2$	${}^1\aleph_3$	${}^1\aleph_4$	${}^1\aleph_5$
<hr/>					
Ω_{15}	\dots	Ω_3	Ω_2	Ω_1	
$\{\Omega_0, \Omega_{-1}\}$					