Here is the differential equation that models the Allomancer/Coin interaction:

$$\frac{d^2y}{dt^2} = \frac{F_p - F_{g,A}}{m_A} + \frac{F_{n,C} - F_p - F_{g,C}}{m_C}$$

Anything on here that is not a constant is a function of time; in controls terminology, these are called 'variables.' Variables are often functions of other variables. There are three kinds of variables:

- Manipulated Variables: the things we can change and *manipulate*. These are 'inputs.'
- Controlled Variables: the things that are affected by other variables, which we want to *control*. These are 'outputs.'
- Disturbances. Things that change with time, which we have no control over and have to deal with. These are also a kind of input.

So, here are our variables:

- y: distance between Allomancer and Coin.
- $\frac{dy}{dt}$: the relative speed between the Allomancer and Coin.
- $F_{n,C}$: the normal force on the coin.
- F_p: the Allomantic force on the coin.
- $F_{p,i}$: the ideal Allomantic force an Allomancer could exert. (This will make sense in a moment.)

$F_{n,C}$ is our disturbance; it is a function of y and t, a step function which magnitude $(F_P + F_{g,C})$ when y<0, added to with a dirac delta function to cancel out the kinetic energy of the coin when it impacts. Obviously, y is our controlled variable. Because the force changes with distance (you can't push as hard when you are farther away), F_p is also a controlled variable. So, we define it as a function of two other variables: the distance y, and the force an Allomancer would exert at point-blank range, $F_{p,i}$.

But what about our other two variables? Model 1 proposes that $F_{p,i}$ is manipulated, and $\frac{dy}{dt}$ is controlled. Model 2 proposes that $\frac{dy}{dt}$ is actually manipulated, and $F_{p,i}$ is controlled. The problem with both of these models is that, for an Allomancer dropping a coin and slowing their descent, there are discontinuities in both $\frac{dy}{dt}$ and $F_{p,i}$ at the same time there is a step function in our disturbance variable. That can only mean one thing….both $\frac{dy}{dt}$ and $F_{p,i}$ are controlled variables. What, then, is our manipulated variable?

Since those two aren't controlled anymore, we can make then functions of whatever we want. Model 5 basically makes them both piecewise functions of a maximum possible speed and a maximum possible force, $\frac{dy}{dt}_{max}$ and $F_{p,i}{}_{max}$. (Subscripts on subscripts! Nice!) This will work mathematically, modelling all the behavior that we've seen Allomancers engage in. The only problem is, it does not share the limitations that we know Allomancers have. The points presented in the earlier writeup holds true, both that Allomancers only have one manipulated variable, and that independently setting both maxima would bypass the danger of being launched back by your own coin.

This is where my model, Model 6, naturally comes in. Both $\frac{dy}{dt}_{max}$ and $F_{p,i}{}_{max}$ are functions of strength S, which is our true manipulated variable. That creates the arbitrary relationship between maximum speed and maximum strength that is the part I don't like about this explanation.