Much has been discussed about the mechanics of Allomancy, how it works with physics. As I recently participated in said discussions, I had become convinced that Brandon used two models for Allomancy, arbitrarily switching between the two. In the following treatise, I will explore several kinematic models, explaining the inconsistencies that they each have with observed instances of Allomancy. At the end, I will present the model that I have managed to create to fit every instance of Allomancy

The Conflicting Instances of Behavior

I have identified two regimes in which Allomancy behaves. In one situation, an Allomancer can be surprised by the amount of force they exert, while in the other one they cannot.

Here is the first situation, when Vin was learning from Kelsier:

[Kelsier] pulled something out of his belt. A clip, the smallest denomination of coin, he held it up before [Vin] standing to the side. “Burn steel, the opposite of the metal you burned a few moments ago.” Vin nodded. Again the blue lines sprung up around her. One of them pointed directly to the coin in Kelsier’s hand. “Alright” Kelsier said, “push on it”. Vin reached toward the proper thread, and pushed slightly. The coin flipped out of Kelsier’s fingers traveling directly away from Vin. She continued to focus on it, pushing against the coin through the air until it snapped against the wall of a nearby house. Vin was thrown violently backward in a sudden jerking motion...”What happened?”, Kelsier asked her. She shook her head, “I don’t know. I pushed on the coin and it flew away, but when it hit the wall I was pushed away.” “Why?” Vin frowned thoughtfully, “I guess...I guess the coin couldn’t go anywhere, so I had to be the one that moved.” “Consequences Vin. You use your own weight when you steel push. If you’re a lot heavier than your anchor it will fly away from you like that coin did. However, if the object is heavier than you are, or if it runs into something that is, you’ll be pushed away. Iron pulling is similar, either you’ll be pulled toward the object, or it will be pulled toward you. If your weights are similar, you’ll both move.”

The second situation is illustrated by when Wax was attempting to hide Marasi’s notebook from the Vanishers, during the wedding:

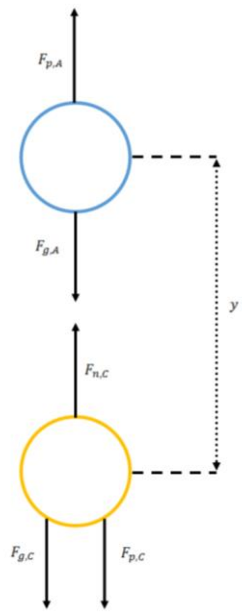
Lines sprang up around [Wax]. One pointed toward the rod, and another to the notebook’s wire coil. He lightly Pushed against them, then let go. The canes and the notebook remained pressed against the table’s side, obscured by the tablecloth, which draped down over them. He had to be careful not to Push too hard, lest he move the table.

There is a third scenario, after the vein of Wax’s illustration, which has similar behavior. I do not have a specific quote from the text, but the situation is this: a Coinshot in midair drops a coin below him. He Pushes against it, sending it to the ground. Once it reaches the ground, the force of the Push increases, but not enough to send him away from the ground, merely to slow his descent towards it.

That is the fundamental question any model for Allomancy must answer. In the second and third situations, even though the object being pushed on cannot move away, the Allomancer did not move away either.

The Equations Governing the Math

Here, I will attempt to lay out the math, going quite simplistic for those who are not very comfortable with physics. The principles here will cover every model presented below, albeit with different values for each of the variables. Here is the free body diagram of an Allomancer (blue circle) and a coin (gold circle), which shows the direction of different forces that act upon several objects. I have drawn it for simplicity's sake with the Allomancer's steelpush aligned parallel to gravity, so as to eliminate any complicated vector math.



There are five forces at work we are concerned with. (Newton's second law, $\Sigma F = ma$, says that adding all the forces together, and then dividing by the mass, lets you find out how an object will accelerate.)

- $F_{g,A}$ and $F_{g,C}$. The force of gravity on an Allomancer and a coin, respectively.
- $F_{p,A}$ and $F_{p,C}$. The force generated by a steelpush that propels an Allomancer and a coin, respectively. These will be equal, but opposite, as long as Newton's third law is not broken.
- $F_{n,C}$. The normal force on the coin while it is in contact with the ground (a.k.a. the force the ground supports to stop the coin from moving). The magnitude of this force is zero when the coin is not on the ground. When the coin is on the ground, the ground will exert whatever force is required to cause the acceleration of the coin to equal zero.

Then, there are the variables used in kinematic equations (the equations that let you use acceleration to calculate velocity).

- There is the distance between the Allomancer and the coin, y.
- The rate of separation, $\frac{dy}{dt}$, is how quickly y is increasing or decreasing. It is basically the relative speed of the Allomancer and the coin. It is determined by the difference in velocities of each object, v_A and v_C. (They are both defined mathematically as upwards velocities; when the coin is launched downward, v_C will be negative.)

- The velocities, in turn, increase or decrease based on the acceleration of each object a_A and a_C. (Also with a similar orientation.)
- The accelerations are found from our helpful force balance above. (Using the masses of the Allomancer and coin m_A and m_C in their respective force balances, of course.)

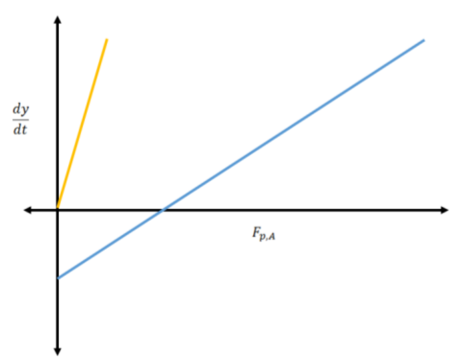
When use these equations, you can relate Forces to acceleration, acceleration to velocity, and velocity to $\frac{dy}{dt}$. When we do this, we can develop a relationship between the rate of separation between an the objects, and the force exerted between them. For the simple situation shown above:

$$\frac{dy}{dt} = v_A - v_C$$

$$= \int a_A dt - \int a_C dt$$

$$= \int \frac{F_{p,A} - F_{g,A}}{m_A} dt - \int \frac{F_{n,c} - F_{p,C} - F_{g,C}}{m_C} dt$$

This will be tricky to solve mathematically, especially since the forces will change with time as the coin and the Allomancer move farther away and interact with their surroundings. But you can clearly see that the presence or absence of the normal force, $F_{n,C}$, will change the resulting relationship between $\frac{dy}{dt}$ and $F_{p,A}$. (When the coin is on the ground, $F_{n,C} = F_{g,C} + F_{p,C}$, so the second term in the above equation will go away entirely.) But, we can approximate what the solutions will look like qualitatively; **the lines below are each a graphical representation of the physics-based relationship, determined by our equations.**

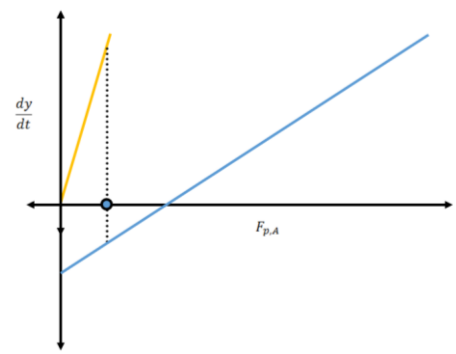


The gold line represents a scenario where $F_{n,C} = 0$; you will be able to get a large displacement from very small force. But the blue line, where a normal force is applied to the coin, the solution shows that when you apply a low force, you do not overcome gravity (and the Allomancer still falls). The blue line crosses the axis when $F_{p,A} = F_{g,A}$. And then, the blue line is overall flatter than the gold line, meaning sthat it takes more force to achieve the desired separation.

So, without further ado, let's look at the various models that can be used to describe steelpushing, and why each of them are wrong.

Model 1: The Allomancer determines a force between himself and an object.

The Allomancer sets the magnitude of $F_{p,A}$ and $F_{p,C}$, which are equal and opposite. (The green dot is what they have power over; the Force of the push.) The harder he pushes, the faster the objects will move away from one another. (You move upward across the graph to until you reach the solution, the $\frac{dy}{dt}$ that is determined by that force applied to that situation.)



This is the plainest explanation, but as poor Vin discovers, it is not a good fit, because that model decouples the Allomancer and the coin.

Problem: Dropping a coin to launch yourself through the air creates a discontinuity in $F_{p,A}$.

If the Allomancer only determins the magnitude of the force, then whatever happens to the coin should not affect the Allomancer. Look at the acceleration of the Allomancer:

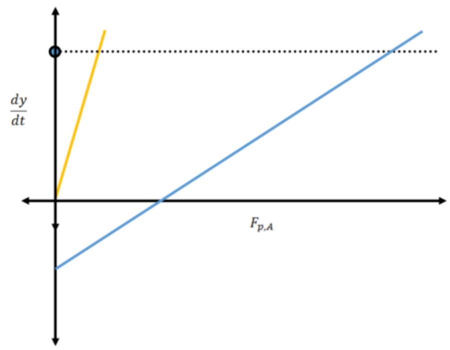
$$a_A = \frac{F_{p,A} - F_{g,A}}{m_A}$$

The coin's acceleration is actually unrelated; the two force balances are decoupled. That's what we see in the graph above; the force that sufficient to launch the coin at a high speed will barely impact the Allomancer, since it's much smaller than the force of gravity on the Allomancer. But that is inconsistent with what we've seen. When a coin contacts the ground, the force on an Allomancer $F_{p,A}$ goes from negligible (much smaller than the force of gravity) to significant (able to overcome gravity to launch them into the air). This is not a conscious decision on the part of the Allomancer to modulate the force of their Push; one of Vin's first lessons was launching a coin against a wall and then being thrown back.

Model 2: The Allomancer determines the rate of displacement between himself and an object.

The variable that is actually controlled in this model is $\frac{dy}{dt}$, the rate at which the coin and the Allomancer separate. The steel line can be viewed as a rigid stick that is growing in length at a determined rate. The force exerted on either end of the stick is still equal and opposite, but the magnitude of that force is dependent on the circumstances. Preservation will provide as much as is needed to keep the Allomancer and the coin moving apart at the desired speed when other forces (like the normal force on a coin) come into play.

Here, the graphical approach is inverted: we start on the y-axis, and move right until we reach the force balance equation:



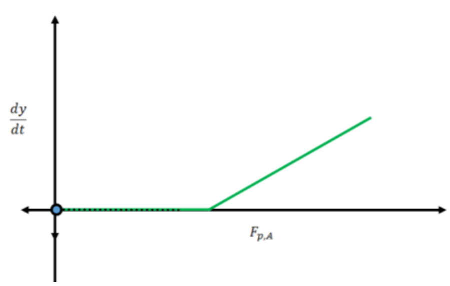
We'll need to do some funny definitions. Coinshots can define $\frac{dy}{dt}$ (referred to here on out as "displacement") as either positive or negative (getting closer or getting farther); they are not limited by the domain (set of possible displacements), but by the range (they cannot generate a force towards themselves).

Problem: Dropping a coin to slow your descent yields a discontinuity in dy/dt.

Imagine you are an Allomancer, dropping through the mists, and you drop a coin below you to Push against to slow your descent. When you drop the coin, you need to define a positive displacement (the coin gets farther away from you) for the entire push, or else you would not be able to send the coin to the ground ahead of you. But once the coin reaches the ground, you will have a negative displacement (even though you are exerting force, the coin is now getting closer to you). This would require an Allomancer to actively changes the nature of his Push when the coin hits the ground (the same problem we had with Model 1).

Problem: During the wedding, Wax specified the force he was pushing with, not the speed he was pushing at.

Here is the graph that would represent the force balance of Wax at the table.



Because there is a normal force on both Wax and on the notebook, it looks slightly different. Wax cannot define a Push at zero displacement; that could correspond to any number of forces. Instead, he definitely defines a force; he makes sure he doesn't push too hard.

Model 3: The mechanism is either direct addition of energy or momentum.

I've seen this idea thrown around a little before, that a force balance is the wrong way to approach the situation. These ideas sidestep issues with the force balance by increasing other attributes of the system.

Problem: A steelpush is possible with the addition of zero kinetic energy and momentum.

Kinetic energy and momentum changes both require a change in velocity. When the Vanishers attacked at the beginning of Alloy of Law, Wax used a slight steelpush to pin Marasi's notebook to the bottom of a table, so the Vanishers couldn't find it. He was adding no kinetic energy to the system, adding no momentum, since nothing was moving. Therefore, we do need to use the force balance.

Model 4: Newton's third law is not obeyed.

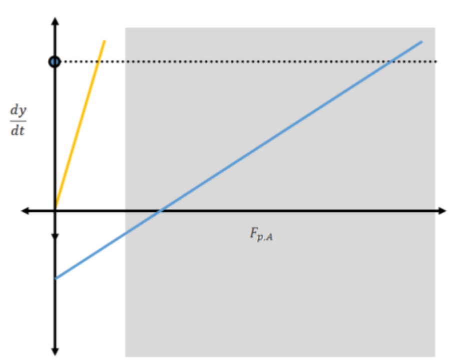
The previous models have all assumed that the force on the coin must be equal and opposite to the force on the Allomancer. But what if Allomancy only does apply force to one at a time? When the effective mass of the Allomancer is larger than the coin, it applies a force to the coin only. When the effective mass of the coin is larger (because it's against the ground), the force is applied to the Allomancer.

Problem: That is an unholy world where any rational analysis will prove futile.

'Effective mass' isn't a meaningful phrase. Qualitatively, it means there's a normal force acting on the object. But mathematically speaking, it requires the push to be determined by information about the coin's surroundings that the Allomancer cannot observe. It also falls to the same issues as Model 1, where the magnitude of the force is different when the coin is being sent down and when the Allomancer is being pushed up. But it cannot be a variation of Model 2, since that math is built around Newtons third law, $F_{p,A} = F_{p,C}$.

Model 5: Models 1 and 2 combined.

This combines the displacement-based equation of Model 2 with the Allomancer's direct control of force from Model 1. When an Allomancer uses a soft push, like Wax did with Marasi's notebook, he defines a positive displacement (make this move away from me) but limits the maximum force that can be exerted. Here is what the graph would look like for when an Allomancer would drop a coin to slow their descent:



The gray area is where they artificially limit themselves; when the coin switches from the yellow line to the blue line, it instead goes as far along the blue line as the Allomancer allows, which just slows their descent ($\frac{dy}{dx}$ is still negative, but not as negative as if there was no force applied).

Problem: Allomancers do not have two points of control over their Push.

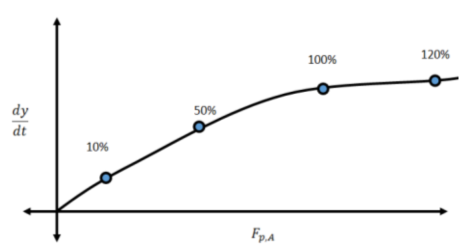
There is no difference between a fast push and a hard push. That would defeat the entire purpose of Vin's first lesson. When fired, a 9mm handgun bullet has enough kinetic energy to kill a person, and the force required to accelerate it to those speeds is not enough to stagger someone firing it. Any coinshot could fire coins with a high displacement and a weak maximum force, and when an opponent coinshot fired back at them, their own Push would never exert enough force to launch themselves backwards. They do not have two ways to control Pushes. Pushing a coin quickly must always open the opportunity to pushing with a large amount of force.

Model 6: The Allomancer determines the "strength" of the Push, which in turn determines the maximum displacement and the maximum force.

Okay, this is the one I think actually works, although I find it inelegant. There are times when only Model 1 (Allomancer defines magnitude of force) matches the descriptions of steelpushing, and there are times when only Model 2 matches the descriptions (Allomancer defines magnitude of displacement). The challenge is finding a mechanism that will simplify to each of those models under different situations, but still maintains the restrictions of each.

The solution that I came up with is that the Allomancer does not define either the force with which they push, or the velocity at which the object is pushed. They define a qualitative "strength" at which they Push, which determines both the maximum displacement they can achieve (how fast they can push it away), and the maximum force they can exert (how hard they can push it away).

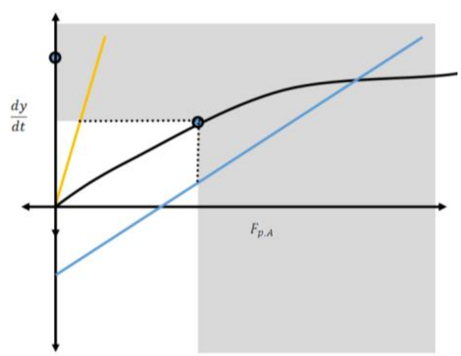
So, there is an artificial relationship between displacement and force; they are both parameters of strength. It's sort of like when you track the motion of a projectile; you can relate the height (x) and the distance (y) in a function y(x). But more realistically, you would also look at each variable as a function of time y(t) and x(t), with time t representing how far along its path it has gone. In this case, the maximum force and the maximum velocity are similarly related through strength, through a parameterized push graph:



Unlike the other graphs we've looked at, **this line does not reflect any physical relationship**. It is not defined from a force balance. It is entirely arbitrary. I made it curvy, just because I was sick of all these straight lines. My impression was that they do not flare steel to push faster, only to push harder, so flaring is pretty much a horizontal regime. But I could be convinced otherwise. Each Allomancer would have a different graph, which could take their weight into account for where their 100% value is along the curve. or maybe some people have differently shaped curves entirely, and are better at shooting things quickly than pushing things powerfully.

The shape of this graph will also be dependent on the distance *y* between the Allomancer and the coin, which is constantly changing. The graph for a different distance will have the points contracted towards the origin of the graph. So, the curve in two-dimensional space is merely a cross-section of a two-dimensional surface in three-dimensional space, with the third axis being y… things get a little messy. I'm not gonna try and graph that for you. We'll just look at instances we're looking at, the parameterized push equation doesn't change much over the distances we're talking about.

So, let's put this one particular parametrized push graph on the same scale as our force balance equations, and see how that turns out:



Since the green dot do not start on an axis, you can move horizontally or vertically to reach the force balance that applies to the physical situation. You can move left, or you can move down; you can't move up (since you can't exceed the maximum displacement), and you can't move right (because you can't' exceed the maximum force). Whether you move left or down, depends on where the graph of the force balance equation lies (which, remember, is the actual physical relationship between $\frac{dy}{dx}$ and $F_{p,A}$.

When there is no normal force (coin is in midair), we are limited by our maximum velocity, and move to the left to reach that force balance for that scenario. When there is a normal force (coin is on the ground), we are limited by our maximum force, and we travel down to reach the force balance.

This gets us back to the original premise, where sometimes Allomancers define their velocity, and sometimes they define their force. They don't actually define either; they define how strong they are Pushing, which then determines both how fast or how strong they can push. Vin can't Push a coin strongly enough to fly through the air without that Push also having the potential to launch her off her feet. Wax may have been able to exert a small Push that wasn't able to move either him or the table, but a Push of that strength against a coin would not have been able to launch it very quickly at all (even if the maximum force threshold could have let it go much quicker).

Problem: There is no good explanation for the maximum displacement and maximum force to have this relationship.

This gets back to observed behavior. To use a coin as a bullet, you do not need very much force at all, but you do need a lot of speed (or else your coin will fall down before it hits your target). Why is a high displacement associated with such a high force in the first place?

I do not believe this is a principle that Brandon created, that he then uses to determine the specific mechanics of Steelpushes. I think he had an idea of what he wanted Steelpushes to do. (If you're not careful, you'll launch yourself through the air. But things get a lot more interesting if you can also use it just as way to more generically apply force. Oh, and Allomancers also need to be able to land, obviously.) The problem is tying them together; when you look at the nitty-gritty math, the simple models are unable to describe every situation. This solution is merely a mathematical construct that allows the behavior we've seen. It lets us essentially switch between Model 1 and Model 2 when a normal force is applied to the smaller object, while still keeping the restrictions that we've seen Allomancers face with regards to being surprised by the strength of a push.

Summary

Okay, how did you do? I hope I didn't lose you along the way. To summarize:

- Allomancers can't just define the force they Push with, because that doesn't match launching yourself through the air with a coin.
- Allomancers can't just define the speed they Push at, because that doesn't match launching a coin down to slow your descent.
- Allomancers can't just limit the maximum force with which they Push, or else Coinshots would not have to worry about being flung backward by an enemy's Push.
- Allomancers can instead define what percentage of their maximum strength they Push at, which in turn determines both their maximum speed and their maximum force.

Here's the key conclusion that allows all the math to work: **When performing Steelpushes, Allomancers cannot reduce the force they would Push against a heavier object, without also reducing the speed at which they would Push a lighter object. This relationship is determined by a particular Allomancer's parameterized push graph.**